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Fractal and Multifractal Approaches to Clustering Phenomena

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Center for Polymer Studies
590 Commonwealth Avenue
Boston University
Boston, Massachusetts 02215
Tel: 617-353-8000
Fax: 617-353-3783

H. Eugene Stanley
H. Eugene Stanley
University Professor
Director, Center for Polymer Studies
Principal Investigator
Tel: 617-353-2617
Fax: 617-353-3783

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I. Introduction and Background

Recently considerable efforts have been made in understanding the dynamics of non-equilibrium interface growth in the context of a variety of fractal clustering models.[1–4] Many recent investigations have concentrated on the dynamic scaling properties of interfaces obtained in experiments and in various cluster growth models. Particular attention has been devoted to the scaling properties of the rms interface width

$$w(\ell,t) \equiv \langle (h(x,t) - \langle h(x,t)\rangle)^2\rangle^{1/2} \sim \ell^{\alpha} f(t/\ell^{\alpha/\beta}). \tag{1}$$

Here $h(x,t)$ is the surface height at time t, the angular brackets denote the average over x belonging to an interval of size ℓ; also, $f(u) \sim u^{\beta}$ for $u \ll 1$ and $f(u) \to Const$ for $u \gg 1$.

It has been widely believed that many such problems belong to the same universality class as the the Kardar-Parisi-Zhang (KPZ) equation,[5]

$$\frac{\partial h(x,t)}{\partial t} = \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \eta(x,t). \tag{2}$$

Here $\eta(x,t)$ is a random noise term. One such model is ballistic deposition,[6] for which the surface width exponents α and β satisfy the general scaling relation[7,8]

$$\alpha + \frac{\alpha}{\beta} = 2 \tag{3a}$$

and can be calculated exactly in the case of normally distributed uncorrelated noise $\eta(x,t)$ in one dimension ($d = 1+1$):

$$\alpha = \frac{1}{2}, \qquad \beta = \frac{1}{3}. \tag{3b}$$

There have recently appeared several experiments on surface growth which yield exponents quite different from those of (3b). For example, in some experiments on immiscible displacement of viscous fluids in porous media it was found that

$\alpha = 0.73 \pm 0.003$ [Ref.9], $\alpha \approx 0.81$, $\beta = 0.65$ [Ref. 10] and, in recent experiments on the growth of bacteria colonies $\alpha = 0.78 \pm 0.06$ [Ref.11].

II. Roughening with Power Law Distributed Noise

Recently Zhang[12] suggested that the anomalous roughening found in experiments can be explained by an uncorrelated "noise" $\eta(x,t)$ obeying a power-law distribution, $p(\eta) \sim \eta^{-\mu-1}$ where $\eta \geq 1$. This anomalous noise can be simulated in a deposition context by depositing rods of size ℓ sampled from a power law probability distribution,

$$p(\ell) \sim \ell^{-\mu-1}. \tag{4}$$

The growth rule is similar to the conventional ballistic deposition rule, i.e., a deposited rod is attached to the highest nearest neighbor surface site. The site at which deposition next occurs can be chosen either *deterministically*[12,13] or *randomly*.[14]

The Zhang idea[12] has received recent support from both mean field theory[15] and numerical simulations,[12–14] both of which suggest that the exponents α and β are anomalously large, and depend continuously on the parameter μ (at least for $\mu < \mu_c \approx 5$).[13–15]

Figure 1 shows the comparison of our results[14] for different values of μ with the Zhang-Krug prediction,

$$\alpha = \frac{3}{\mu+1}, \qquad \beta = \frac{3}{2\mu-1}, \qquad [\mu \geq 2]. \tag{5}$$

It can be seen that for $\mu \geq \mu_c = 4.5 \pm 0.5$ both α and β are almost independent of μ and are very close to the classical values $\alpha = 1/2$, $\beta = 1/3$. For $\mu < \mu_c$ both exponents deviate from their "classical" values and approach the limiting values $\alpha = 1$, $\beta = 1$ predicted by the Zhang-Krug relation for $\mu = 2$. As a final consistency check, we found excellent agreement with the scaling relation $\alpha + \alpha/\beta = 2$ in the entire range of studied values of $\mu \geq 2$. For $\mu < 2$, see our discussion below.

In a recent work[16] a closed-form expression for the probability distribution for the fluctuations $\delta h(x,t) \equiv h(x,t) - \langle h(t) \rangle$ in surface height $h(x,t)$ was suggested based on a formal analogy between anomalous roughening and the statistics of a Lévy walk.

By a Lévy walk we mean that each unit of time a random walker steps a unit length. The walker moves ℓ successive steps in the same direction before randomly changing direction, and ℓ is taken from a Lévy distribution[17] $p(\ell) \sim \ell^{-\mu-1}$. The probability density $P(x,n)$ that the walker is at position x after n steps has a tail distribution ($x \gg n^{1/\mu}$) of the form[18]

$$P(x,n) \sim n/x^{\mu+1} = \frac{1}{n^{1/\mu}} \left(\frac{x}{n^{1/\mu}}\right)^{-\mu-1}. \tag{6}$$

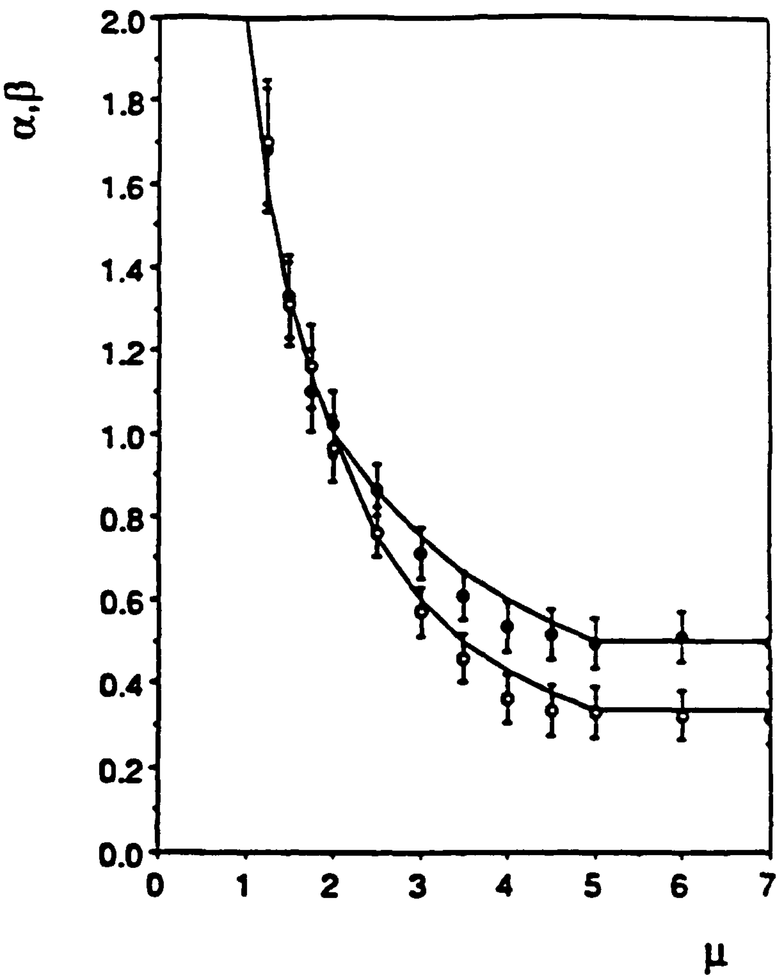

Fig. 1. Comparison of numerical results for exponents α (•) and β (○) with theoretical predictions given for $\mu \geq 2$ by Eqs. (5) (solid line), and for $\mu \leq 2$ by Eq. (10) (solid line). After Ref. 16.

By mapping the surface growth to Levý walk we obtain[16] that for $t \gg t_\times \sim L^z$ the distribution of $h(x,t)$ has the asymptotic form

$$P(\delta h, L, t \gg t_\times) \sim \frac{1}{L^{(3-\alpha)/\mu}} \left(\frac{\delta h}{L^{(3-\alpha)/\mu}}\right)^{-\mu-1} \qquad [\delta h \gg L^{(3-\alpha)/\mu}]. \tag{7}$$

Since for $t \gg t_\times$, $\delta h \sim w \sim L^\alpha$, we find a self consistent equation for α, $\alpha = (3-\alpha)/\mu$, or $\alpha = 3/(\mu+1)$, the same as Eq. (5). Note that α assumes its classical value $\alpha = 1/2$ at $\mu = \mu_c = 5$. Substituting $\alpha = 3/(\mu+1)$ into (7), we obtain the predicted scaling form

$$P(\delta h, L, t \gg t_\times) \sim \frac{1}{L^{3/(\mu+1)}} \left(\frac{\delta h}{L^{3/(\mu+1)}}\right)^{-\mu-1}. \tag{8}$$

To test (8), we performed simulations of ballistic deposition using Eq. (4) for several values of μ and for a sequence of values of L.[16] Figure 2 supports the predicted data collapse of Eq. (8) for $\mu = 1.5$ and 3.

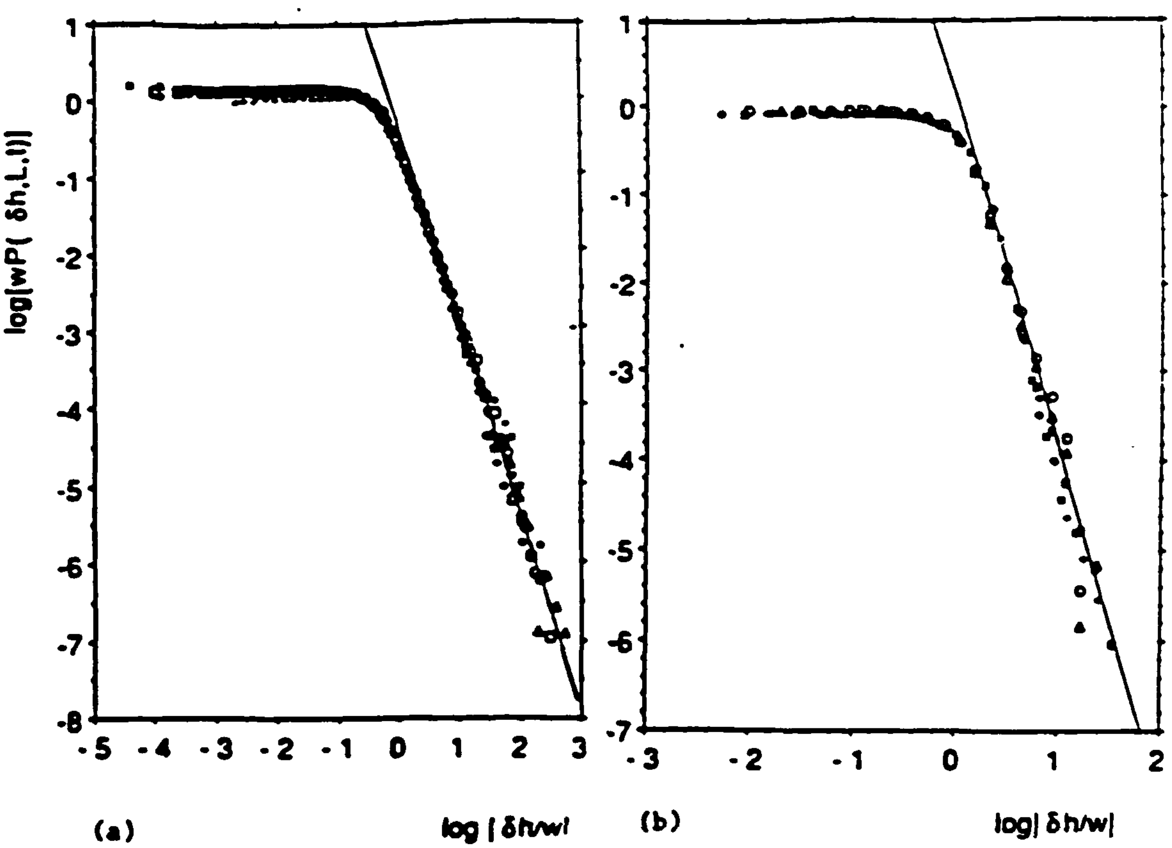

Fig. 2. Log-log scaling plot of $wP(\delta h, L, t)$ against $|\delta h|/w$. Here w is the *first* moment of $P(\delta h, L, t)$ for (a), $\mu = 1.5$ and for (b), $\mu = 3$. Symbols in (a) are: $+$ ($L = 1028, t = 64$); $\diamond$ ($L = 2048, t = 256$); $\times$ ($L = 2048, t = 1024$); $\triangle$ ($L = 256, t > 4096$); $\square$ ($L = 512, t > 4096$); $\circ$ ($L = 1024, t > 4096$). Symbols in (b) are: $\square$ ($t = 32, L = 2048$); $\triangle$ ($t = 128, L = 2048$); $\circ$ ($t = 512, L = 2048$); $+$ ($t > 16384, L = 512$); $\times$ ($t > 16384, L = 1024$); $\diamond$ ($t > 32768, L = 2048$). The straight lines have slopes of (a) 2.5 and (b) 4, as predicted by Eq. (7). After Ref. 16.

To obtain the early time ($1 \ll t \ll t_\times$) dependent probability we use again the time-space scaling relation $t \equiv L^{\alpha/\beta}$ of (1) and L in (7) should be replaced by $tL = t^{1+1/z}$ yielding

$$P(\delta h, t) \sim \frac{1}{t^{(z+1)/\mu z}} \left(\frac{\delta h}{t^{(z+1)/\mu z}} \right)^{-\mu-1}. \tag{9}$$

Since $\delta h \sim t^\beta$ we obtain $\beta = (z+1)/\mu z = 3/(2\mu - 1)$, the same as in (5). Figure 2 shows also data for the time-dependent probability density supporting (9).

The above considerations, Eqs. (7)-(9), are valid for $\mu > 2$. For $\mu \leq 2$, the relation $t_\times L \sim L^{z+1}$ fails since $t_\times$ is bounded from below by L. Thus one must repeat the arguments leading to Eqs. (7)-(9) using $t_\times L = L^2$ from which follows[16]

$$\alpha = \beta = \frac{2}{\mu}, \qquad [\mu < 2]. \tag{10}$$

Note that (10) complements the Zhang-Krug prediction (5) for values of μ below 2. Numerical data supporting (10) are shown in Fig. 1. The analogy

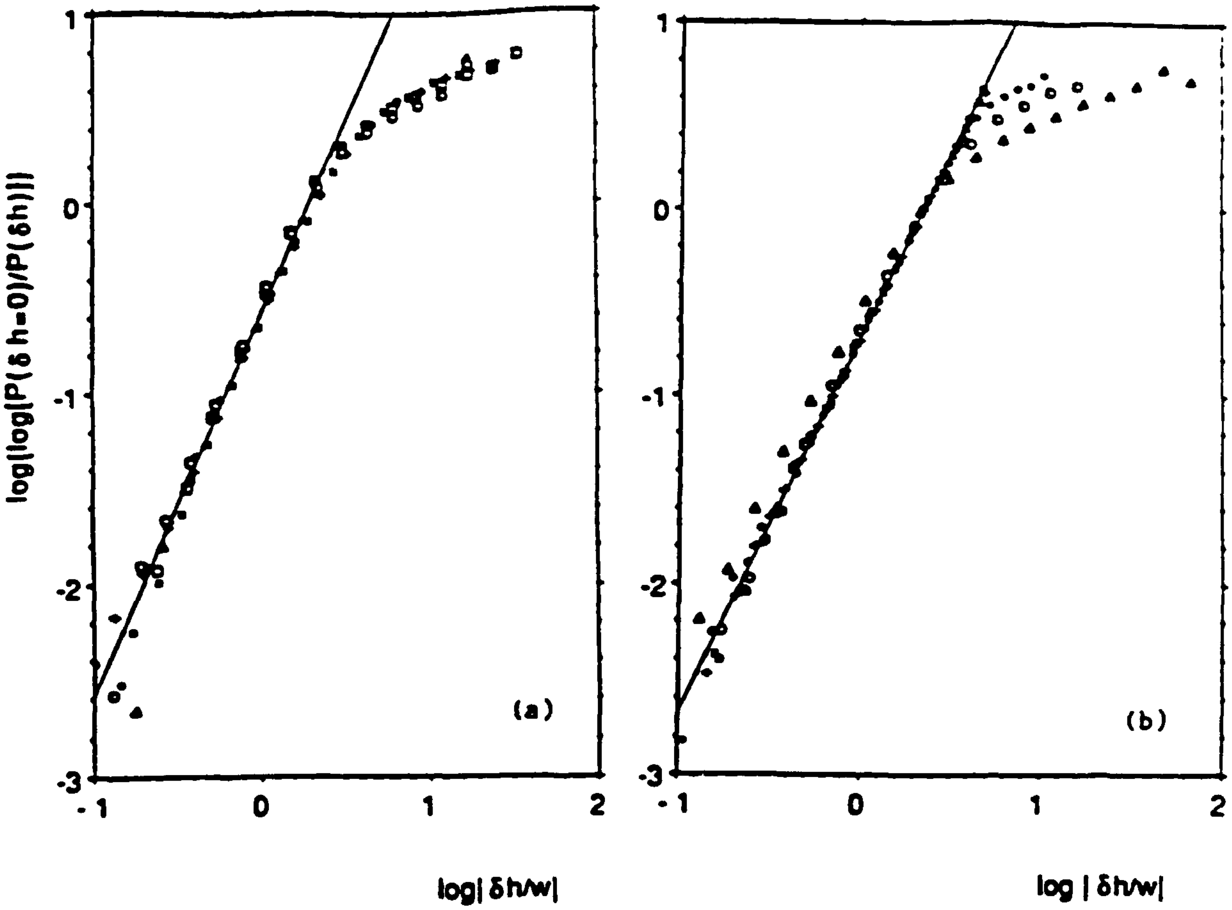

Fig. 3. (a) Log-log scaling plots of $\log[P(\delta h = 0, L, t)/P(\delta h, L, t)]$ versus $|\delta h|/w$ for $\mu = 3$ for different system sizes L and times t. The straight line has the slope 2, as should be for Gaussian distribution. The symbols are the same as those used in Fig. 2(b). (b) Log-log plot of $\log[P(\delta h = 0, L, t)/P(\delta h, L, t)]$ as a function of $|\delta h|/w$ for different values of μ (the data for each value of μ obtained for $L = 1024$ and large t, greater than $t_\times$): □ ($\mu = \infty$); × ($\mu = 6$); + ($\mu = 5$); ◇ ($\mu = 4$); ○ ($\mu = 3$); △ ($\mu = 2$). The crossover value of $\delta h/w$ at which the behavior changes from Gaussian (straight line with slope 2) to power law increases gradually with the value of μ. After Ref. 16.

to Lévy walks predicts not only the tails of the probability densities but also their behavior in the range of small fluctuations. In this range and for $\mu \geq 2$ the distribution of Lévy walks is known to be Gaussian,[18] predicting that for $\delta h < w$ the probability density of surface heights is also Gaussian. Indeed, plotting the data in Fig. 3 as $\log\{\log[P(\delta h, L)/P(0, L)]\}$ versus $\log \delta h$ for several values of L shows a clear range of slope 2 supporting a Gaussian form. The crossover from Gaussian to a power-law occurs at a value of $y = \delta h/w$ which increases as μ increases as expected from the analysis of theory in Ref. 18.

As seen from Fig. 2, both time and size dependence have the *same* scaling relation. Indeed, Eqs. (8) and (9) and the Gaussian form found for small fluctutions can be combined for $\mu \geq 2$ to a single scaling relation

$$P(\delta h, L, t) \sim \frac{1}{w} F\left(\frac{\delta h}{w}\right), \tag{11a}$$

where $w = w(L,t)$ is given by Eq. (1), and

$$F(y) \sim \begin{cases} \exp(-ay^2) & [y < y_c] \\ y^{-\mu-1} & [y > y_c]. \end{cases} \tag{11b}$$

The data collapse shown in Fig. 2 supports (11). Similar numerical results, supportng (11) for $\mu = 3$ were presented in Ref. 13.

From the analogy to Lévy walks we expect that in the case of conventional ballistic deposition ($\mu = \infty$) the distribution will be Gaussian:

$$P(\delta h, t) \sim \frac{1}{t^{1/3}} e^{-a(\delta h)^2/t^{2/3}} \tag{12a}$$

for $t \ll t_\times$, and

$$P(\delta h, L) \sim \frac{1}{L^{1/2}} e^{-a(\delta h)^2/L} \tag{12b}$$

for $t \gg t_\times \sim L^{1/z}$. In Fig. 4 we show numerical data supporting (12).

The Zhang[12] model, which is based on the assumption that the noise in the system has power law distributed amplitudes, may be the explanation for the anomalous surface roughening in experiments but the origin of such a noise in real systems remains unclear.[19]

III. Correlated Noise

Anomalous surface roughening can be also due to long-range correlated noise. Next we review recent studies on the effect of long-range correlated noise on surface growth models.

When the noise itself is the result of another stochastic process, then the noise *cannot be treated as random*—the noise is correlated in space and/or time.[20] In this case, the exponents depend on the strength of the correlation. Medina et al[5] used dynamical renormalization-group analysis to study the KPZ equation with long range correlated noise. The noise they studied has the correlation

$$\langle \eta(\mathbf{x},t)\, \eta(\mathbf{x}',t') \rangle \sim |\mathbf{x} - \mathbf{x}'|^{2\rho-(d-1)}\, |t - t'|^{2\theta-1}, \tag{13a}$$

where d is the overall dimension of the system ($d-1$ is the dimension of the surface). If the noise has no temporal correlation, i.e.,

$$\langle \eta(\mathbf{x},t)\, \eta(\mathbf{x}',t') \rangle \sim |\mathbf{x} - \mathbf{x}'|^{2\rho-(d-1)}\, \delta(t - t'), \tag{13b}$$

the exponents obey the relation $\alpha + z = 2$. Since then there is only one independent scaling exponent, it is sufficient to give β; for $d = 1+1$

$$\beta = \begin{cases} 1/3 & 0 < \rho \le \frac{1}{4} \\ (1+2\rho)/(5-2\rho) & \frac{1}{4} < \rho \le 1. \end{cases} \tag{14a}$$

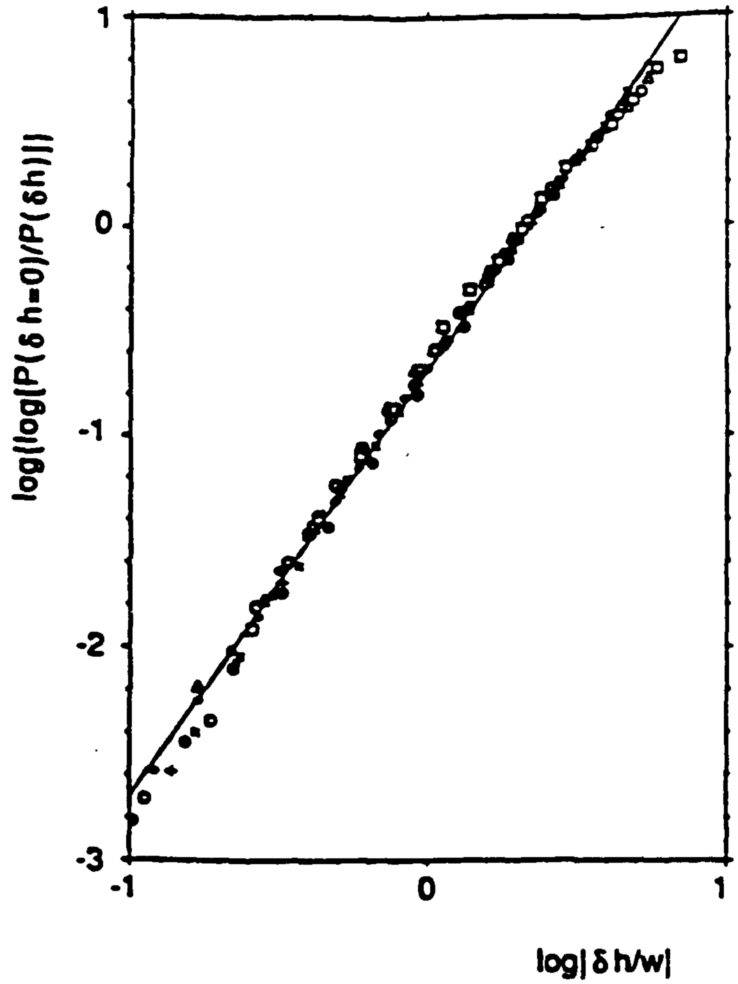

Fig. 4: Log-log plot of $\log[P(\delta h = 0, L, t)/P(\delta h, L, t)]$ as a function of $|\delta h|/w$ for $\mu = \infty$, which is the case of *conventional* ballistic deposition. Here w scales according to Eq. (1) with $\alpha = 1/2$, $\beta = 1/3$. The Gaussian behavior is found in the entire range of δh: $\square$ ($t = 64, L = 4096$); $\triangle$ ($t = 256, L = 4096$); $\circ$ ($t = 1024, L = 4096$); $+$ ($L = 512, t > 16384$); $\times$ ($L = 1024, t > 16384$); $\diamond$ ($L = 2048, t > 32768$); $\bullet$ ($L = 4096, t > 32768$). After Ref. 16.

The other feature of the KPZ equation is that it can be mapped to the directed polymer (DP) problem[21]. The noise plays the role of a time-dependent random potential. Thus, the results of Ref. 5 can also apply to the DP problem in a correlated potential field.

Zhang[22] used a replica method to study the DP problem with correlated noise η given by Eq. (13b). Due to the analogy between the DP problem and the KPZ equation, Zhang predicts for $d = 1 + 1$

$$\beta = \begin{cases} (1+2\rho)/(3+2\rho) & 0 < \rho \leq \frac{1}{2} \\ (1+2\rho)/(5-2\rho) & \frac{1}{2} < \rho \leq 1. \end{cases} \tag{14b}$$

Hentschel and Family[23] studied the scaling behavior for dissipative dynamical systems and proposed a new relation:

$$\beta = \frac{1}{3-2\rho}, \qquad 0 \le \rho \le \frac{1}{2}. \tag{14c}$$

Note that the three predictions [Eqs. (14a), (14b), and (14c)] differ for $0 < \rho < 1/2$.

There have been several prior attempts to verify the analytical results with correlated noise.[24] This work relies on numerical methods that probably generate undesired correlations in the noise. Here we review a recent work[25] where we generate algebraically-correlated noise,[26] integrate numerically the KPZ equation, and also simulate the DP growth in a correlated potential field. The results of Peng et al.[25] for both KPZ and DP agree with each other, and qualitataively agree somewhat better with (14c) than with (14a) or (14b). Finally we implement correlated noise into the BD model, and were surprised to find surface roughening exponents that differ from *both* the KPZ equation *and* the DP problem.

To construct the algebraically-correlated noise, we first generate a representation of random Gaussian uncorrelated noise $\eta_o(\mathbf{x}, t)$, then Fourier transform it to obtain $\eta_o(\mathbf{q}, \omega)$. We define

$$\eta(\mathbf{q}, \omega) \equiv |\mathbf{q}|^{-\rho} \, |\omega|^{-\theta} \, \eta_o(\mathbf{q}, \omega). \tag{15}$$

The noise $\eta(\mathbf{x}, t)$ is obtained by Fourier transforming $\eta(\mathbf{q}, \omega)$ back into the space and time domain. It is straightforward to verify that $\eta(\mathbf{x}, t)$ obtained in this way has the correct correlations (13a). We restrict ourselves to the $d = 1 + 1$ case and the noise has only spatial correlation ($\theta = 0$) as in Eq. (13b).

(i) Consider first the KPZ equation with noise η described by (13b). For a one-dimensional surface, the discrete form of Eq. (2) is

$$\begin{aligned} h_{t+\Delta t}(i) = h_t(i) &+ \Delta t[h_t(i+1) + h_t(i-1) - 2h_t(i)] \\ &+ \frac{\lambda \Delta t}{2} \left[\frac{h_t(i+1) - h_t(i-1)}{2} \right]^2 + \sqrt{\Delta t}\eta_t(i). \end{aligned} \tag{16}$$

Small Δt is needed to obtain good convergence, and we choose the appropriate time step by verifying that smaller time steps do not change our results. We obtain the exponent β from $w(\ell, t)$ defined in Eq. (1), since $w \sim t^\beta$ for $\Delta t \ll t \ll t_\times$.

We start with the case $\lambda = 0$ (no non-linearity) for which z and β can be found exactly from dimensional analysis[2,27]: a change of scale $x \to bx$ and $t \to b^z t$ implies $h \to b^\alpha h$ and

$$\eta(x, t) \to b^{\rho - 1/2 - z/2} \eta(x, t)$$

[from Eq. (13b)]. Equation (2) is scale invariant for the choice

$$z_0 = 2 \quad ; \qquad \beta_0 = \frac{1}{4} + \frac{\rho}{2}. \tag{17}$$

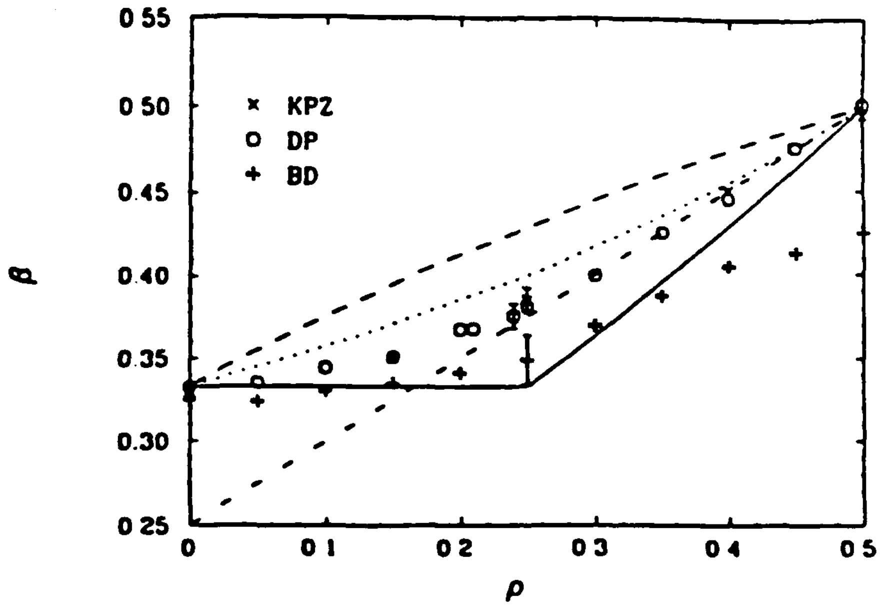

Fig. 5. Comparison of our numerical results and theoretical predictions of (14a), (14b), and (14c) (solid, dashed, and dotted lines respectively). Typical error bars are shown for each of the three models treated. The dot-dashed line, Eq. (17), is obtained by neglecting the non-linear term in Eq. (2). After Peng et al., Ref. 25.

Our numerical simulation for $\lambda = 0$ confirms (17).

When $\lambda \neq 0$ the exponents change. We find that the exponent β approaches the same value for non-zero λ. Since changing λ should not change the universality class, we carry out our simulation for that value of λ which gives the fastest convergence to the correct value of β; then we vary the parameter ρ. The results are shown in Fig. 5. The solid, dashed, and dotted lines are the predictions from three theories [Eqs.(14a), (14b), and (14c)], respectively.

To check our results, we also study the DP growth. By a simple transformation $W(\mathbf{x},t) \equiv \exp[(\lambda/2)h(\mathbf{x},t)]$, we obtain from (2)

$$\frac{\partial W}{\partial t} = \nabla^2 W + \frac{\lambda}{2}\eta(\mathbf{x},t)W. \tag{18}$$

Here W is the sum of Boltzmann weights for all configurations of a DP connecting $(\mathbf{0},0)$ and $(\mathbf{x},t)$, and $\eta(\mathbf{x},t)$ is the potential field. The Boltzmann weight for all paths joining the points $(0,0)$ and (x,t) is

$$W(x,t) \equiv \sum_c \exp[-E_c/kT]. \tag{19}$$

Here E_c is the sum of the potential field η on configuration c, and the sum is over all configurations joining the two end points.

The typical transverse fluctuation scales with the length of the polymer t as $\langle x^2(t)\rangle^{1/2} \sim t^\nu$. At zero temperature, only the optimal path (configuration with minimum energy) makes a contribution. Since the optimal path still dominates at finite but low temperature, we choose $T = 0$ to simplify our numerical task.

We generate a representation of $\eta(x,t)$ [obeying Eq. (13b)], and record the end point of the optimal path $x(t)$. We average over many realizations (typically 10^5 of $\eta(x,t)$). The exponent ν is related to the dynamic exponent $z = \alpha/\beta = 2-\alpha$ of KPZ equation via $\nu = 1/z$. Hence to compare with the KPZ results, we define $\beta_{\rm DP} \equiv 2\nu - 1$ and show the results in Fig. 5. The agreement with our numerical results for the KPZ equation provides an excellent consistency check on our numerical methods.

(ii) Next we study the BD model with algebraically spatial correlated noise. For *uncorrelated* BD,[1-4] particles rain down vertically onto the substrate until they reach one of the growth sites. A growth site is defined as the highest site on each column that belongs to the nearest neighbors of the deposition surface. Once the particle reach the growth site it stops and become a part of the deposit. Note that the deposition rule defined above allows lateral growth, which is believed to be described by the non-linear term $[(\nabla h)^2]$ in Eq. (2).

We introduce correlated noise according to (13b). As seen from Fig. 5, we find significant differences between exponents obtained from the BD model and the DP growth (or the KPZ model).

IV. Experiment and a Directed Percolation Model

Next we present experiments in which ink, coffee and other suspensions are absorbed by a hanging paper, forming a rough interface between wet and dry regions. We analyze this morphology and measure its roughness exponent α, Eq. (1). Based on the experiment we propose a new model for interface roughening. Both the model and the experiment produce interfaces with an anomalously large value of α.[28]

(a) Experiment. The experiment was performed by clipping paper to a ring stand, and allowing it to dip into a basin filled with suspensions of ink or coffee. The suspension was absorbed into the paper, forming a rough interface between the wet and the dry regions. We allow the interface to rise until it stops and no change in either height or shape of the interface is observed. The stopping can be attributed to the evaporation of the fluid in the wet regions. After drying, we digitize this rough interface. We then calculate the height-height correlation function[10] $c(\ell,0) \equiv w(\ell)$ on different length scales ℓ, averaging over ten different interfaces. Figure 6a shows the data, which support a scaling of the form $c(\ell,0) \equiv w(\ell) \sim \ell^{\alpha}$ with $\alpha = 0.63 \pm 0.04$.

(b) Model. The model we propose is defined as follows: on a square lattice of edge L (with periodic boundary conditions) we block a fraction p of the cells to correspond to the inhomogeneous nature of the paper towel. At $t=0$, we regard the "interface" to be the bold horizontal line shown in Fig. 7a. At $t=1$ we randomly choose a cell (labeled X in Fig. 7b) which is one of the unblocked dry cells that are nearest neighbors to the interface. We wet cell X and *any cells that are below it in the same column.* This process is then iterated. For

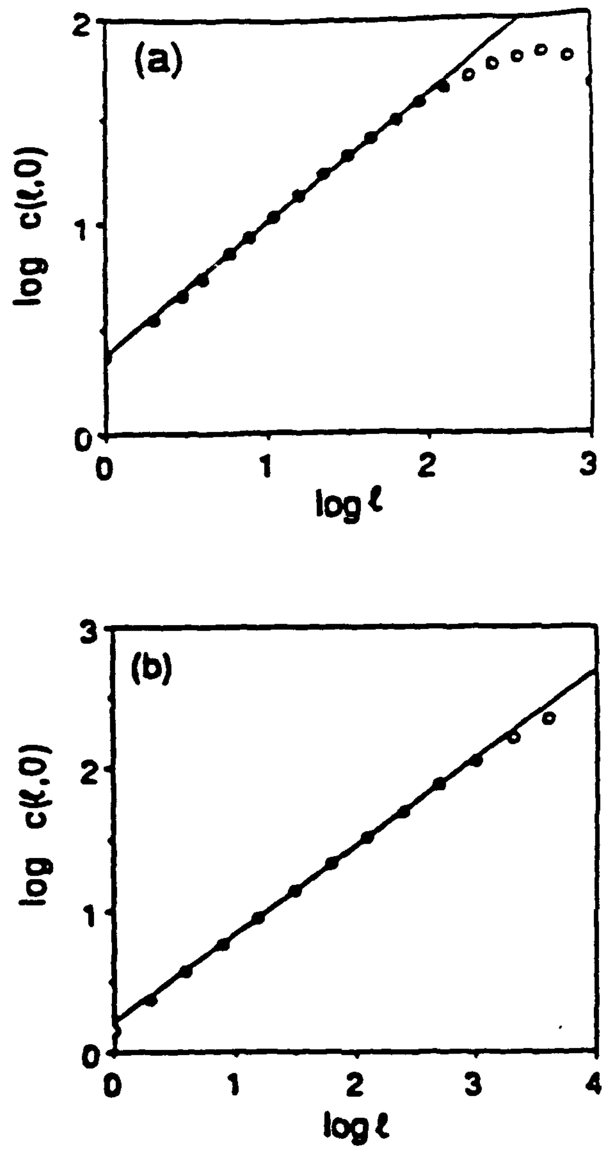

Fig. 6. Log-log plots showing the dependence on length scale ℓ of the height-height correlation function $c(\ell, 0) \sim w(\ell)$ for (a) the experimental data (averaging over 10 different experiments), and (b) the numerical results (averaging over 1000 different realizations for system size $L = 16,384$ and for $p = 0.4675$, very close to p_c for the infinite system). The slope for the set of experimental points indicated by solid circles (two decades) is 0.63 ± 0.04, while the slope for the simulation point indicated by solid circles (three decades) is 0.63 ± 0.02. After Barabási et al. Ref. 28.

example, Fig. 7c shows that at $t = 2$ we choose cell Y a second unblocked cell to wet, while Fig. 7d shows that at $t = 3$ we wet cell Z *and also cell Z′ below it.*[29]

We find that for p below a critical threshold[30] $p_c = p_c(L)$ the interface propagates without stopping, while for p above p_c the interface is pinned. Figure 6b displays the scaling behavior of the model at criticality, and we find that $\alpha = 0.63 \pm 0.02$, a value identical to the experimental value of Fig. 6a.

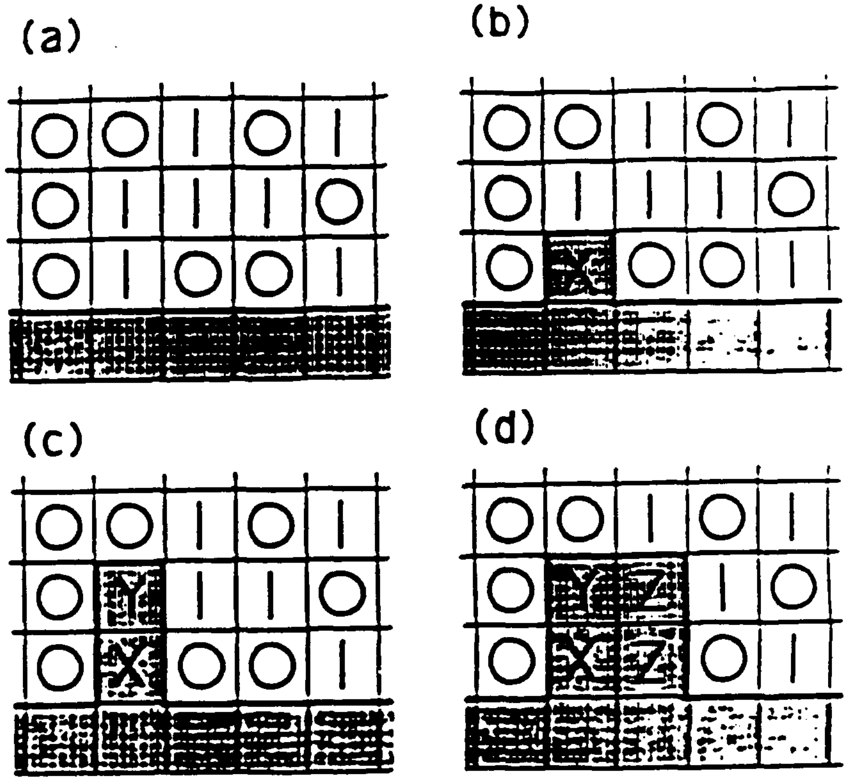

Fig. 7. Explanation of the model for interface growth with erosion of overhangs. Wet cells are indicated by shaded cells. Dry cells are randomly blocked with probability p (indicated by 0 or unblocked with probability $1-p$ (indicated by 1). The interface between wet and dry cells are shown by a heavy line. (a) $t = 0$, (b) $t = 1$, (c) $t = 2$, and (d) $t = 3$. After Barabási et al., Ref. 28.

Next we argue that the model presented above is connected to directed percolation,[4,31] thereby providing a theoretical basis for the observed and calculated values of the anomalous roughening exponent α. The propagation of the interface will stop when it reaches for the first time a directed path of blocked cells leading from West to East—this path is such that one can walk on it from West to East without turning to the West. Such a 'directed path' is a path on the directed percolation cluster formed by the cells labeled 0. We assume that a single transverse length characterizes the directed percolation clusters so that the width w of this interface scales as the transverse correlation length $\xi_\perp$ of the directed percolation problem ($\xi_\perp$ is a rigorous upper bound). Thus we assume $w(\ell) \sim \xi_\perp$ and $\ell \sim \xi_{||}$, where $\xi_{||}$ is the longitudinal correlation length in the corresponding directed percolation problem. Since $\xi_\perp \sim \xi_{||}^{\nu_\perp/\nu_{||}}$ we identify[4,31,32] $\alpha = \nu_{||}/\nu_\perp \simeq 0.63$.

To probe the *dynamics* of the growing interface in the model, we study the height-height correlation function $c(\ell, t)$. Our numerical results support an exponent $\beta = 0.64 \pm 0.04$. The usual exponent identity attributed to the Galilean invariance, which is known to be valid for the KPZ equation,[5] is violated; we find $\alpha + z \simeq 1.69$, smaller than two. This is a consequence of the

strong anisotropy of the mechanism which excludes the overhangs: An infinitesimal tilting of the pinned interface will result in removing blocked cells, thus allowing the interface to propagate further.

Further support for the directed percolation model can be obtained if we consider a finite system at a fixed value $p_o < p_c$. If $\xi_{||}(p_o)$ is larger than the system size L, the interface may be stopped by the directed percolation path. Thus we identify two regimes: Regime I where $\xi_{||} > L$ and Regime II where $\xi_{||} < L$. In Regime I, we observe only anomalous roughening ($\alpha \simeq 0.68$), while in Regime II we predict a crossover to behavior described by the KPZ exponent ($\alpha = 0.5$). A similar crossover[33] is observed, both in our calculations and even in some recent experiments (Fig. 3 of Ref. 10).

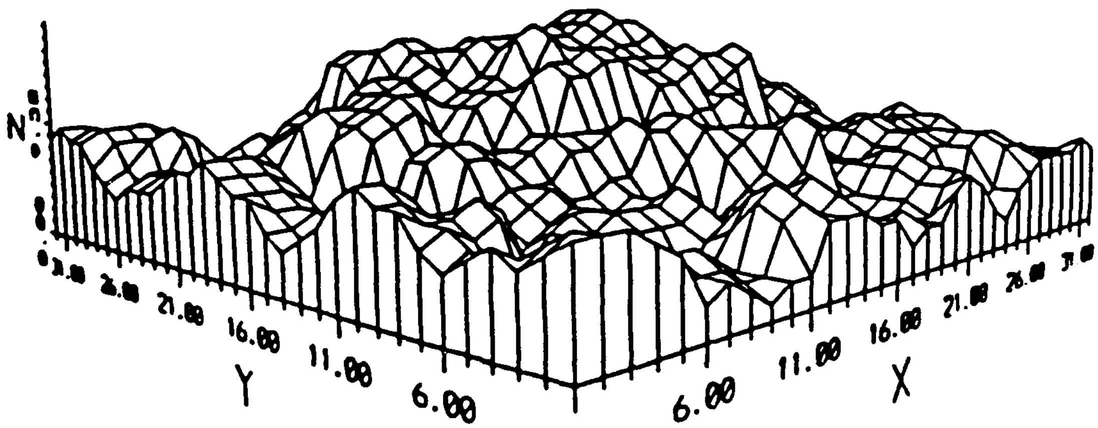

Fig. 8. A two-dimensional "directed surface" with erosion of overhangs using the rules described in Fig. 7. The critical probability of blocked sites is $p_c \simeq 0.74$.

We also studied several variants of the above model. One interesting variant arises if we replace the blocked-unblocked percolation substrate by one used in invasion percolation. At every cell of the lattice we put a random number between 0 and 1, and advance the interface to the nearest neighbor with the smallest random number. We use the same mechanism as in the previous model to erode the overhangs. In this model the interface never stops propagating unless we introduce a cutoff (a particular value of $p = p_1$; cells possessing a random number $p > p_1$ are blocked cells). Thus the interface that stops propagating is generated by the same mechanism as in the normal percolation model, i.e. they are in the same universality class. However, one significant difference is that the normal percolation interface is generated basically by a local growing rule, while in the invasion percolation the growing rule is global. Numerical studies on the *moving* interface—both of invasion and normal percolation substrate—give the same roughness exponent, $\alpha = 0.68 \pm 0.05$, slightly larger than for the pinned interface.

Barabási et al [28] also studied the above model in higher dimensions, for which there are no theoretical predictions on the values of the scaling exponents (see Fig. 8). Our results suggest $\alpha = 0.51 \pm 0.05$ for $2 + 1$ dimensions, which

is larger than the most accurate available numerical result[34] for KPZ growth $\alpha = 0.4 \pm 0.01$. Using similar arguments as for $d = 1+1$, we obtain here a two-dimensional "directed surface" with correlation exponents $\nu_{\|}$ and $\nu_{\perp}$, which obey $\alpha = \nu_{\|}/\nu_{\perp} = 0.51$.

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PATENTS FILED

NONE

PATENTS GRANTED

NONE

HONORS, INVITED TALKS AND PUBLICATIONS IN GRANT PERIOD

AWARDS & HONORS RECEIVED DURING THE GRANT PERIOD

(1) Thirtieth Saha Memorial Lecture, 1991 (delivered 5 Jan. 1992). [half the previous lecturers are Nobelists]

(2) December 1990: Science Citation Index recognition for Top 100 most-cited articles of 1988 [for A. Aharony, R. J. Birgeneau, A. Coniglio, M. A. Kastner and H. E. Stanley, "Magnetic Phase Diagram and Magnetic Pairing in Doped La_2CuO_4," Phys. Rev. Lett. **60**, 1330-1333 (1988).].

(3) Member, International Scientific Organizing Committee, *IUPAP International Conference on Statistical Mechanics (STATPHYS-17)*, Rio de Janeiro, August 1989.

(4) Co-Director, NATO Advanced Study Institute, *Propagation of Correlations in Constrained Systems*, Cargèse, France, 2-14 July 1990.

(5) Co-Director, International workshop on *Random Materials & Processes*, University of the West Indies, 18-22 December 1990.

(6) Member, International Scientific Organizing Committee, *IUPAP International Conference on Statistical Mechanics (STATPHYS-18)*, Berlin, August 1992.

INVITED TALKS DURING THE GRANT PERIOD

(1) Invited Talk, *Complexity in Physics: Entering the 21st Century*, Stockholm, Sweden, 3-8 September 1990.

(2) Opening Talk, *Random Materials & Processes*, University of West Indies, 18-23 December 1990.

(3) Opening Talk, *American Physical Society Sectional Meeting*, Smith College, 5-6 April 1991.

(4) Invited Talk, *American Chemical Society*, Atlanta, GA, 14-19 April 1991

(5) Invited Talk, *International Workshop on Physics of Inhomogeneous Materials*, International Center of Theoretical Physics, Trieste, Italy. 10-15 June 1991.

(6) Invited Talk, International Conference *Teaching Fractals and Chaos*, Chubu Univ, Nagoya, Japan, 19-24 August 1991.

(7) Invited Talk, NATO Advanced Research Workshop *Growth Patterns in Physical Sciences and Biology*, Granada, Spain, 7-11 October 1991.

(8) Opening Address, Molecular Simulations Symposium, *American Physical Society*, Richmond, VA 13-15 Nov 1991 [talk delivered by S. Buldyrev]

(9) Invited Talk, *International Symposium on Phase Transitions*, Tel Aviv University, 26-27 December, 1991.

(10) Invited Talk, International Conference on Random Systems, Calcutta, 27 December 1991-7 January 1992

(11) S. N. Bose Memorial Lecture, Calcutta, January 1992.

(12) Thirtieth Saha Memorial Lecture, Calcutta, January 1992.

(13) Opening lecturer, Middle Europe Conference on Statistical Mechanics, Univ. of Belgrade, Yugoslavia. 31 March - 2 April 1992.

(14) Invited Lecturer, International Workshop *Surface Disordering: Growth, Roughening and Phase Transitions*, Les Houches, France, 31 March-9 April 1992.

(15) Invited Lectuer, International Conference on *Water-Biomolecule Interactions*, Palermo, Italy 1-4 June 1992.

(16) Invited Lectuer, *66th Colloid and Surface Science Symposium*, American Chem. Society, Morgantown, WVA. 14-17 June 1992

(17) Keynote Address, International Conference on Fractals and Disordered Systems, Hamburg, Germany, 27-31 July 1992

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